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MODEL ANALYSIS OF THE
PATHFINDER BOILING WATER REACTOR

by

Francis Vito Rossano

A Thesis Submitted to the
Graduate Faculty in Partial Fulfillment of
The Requirements for the Degree of
MASTER OF SCIENCE

Major Subject: Nuclear Engineering

Signatures have been redacted for privacy

Iowa State University
Of Science and Technology
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I. INTRODUCTION

The use of physical models has been useful in the nuclear engineering field (1, 2, 3, 10, 12, 13, 22). These models have provided data to reinforce the results of analytical studies, and to provide engineering design data where analytical studies are too difficult. These small models are generally cheaper and easier to build and operate than are the larger full size mock-ups.

To date, most of the thermal hydraulic modeling in Nuclear Reactor Engineering work where the size of the model is significantly smaller than the prototype has been limited to pressure drop, flow distribution and mixing studies under isothermal conditions (1, 2, 3, 10, 12, 13). The only thermal model maintained similarity by keeping the length scale unity, and the system parameters the same as in the prototype (22). It is the object of this analysis to find an approach to meeting the similitude requirements for a thermal hydraulic model with a length scale other than unity. The Pathfinder superheater will be modeled to reach this goal.

First, all possible parameters that enter into the Pathfinder thermal hydraulic system will be studied to see if exact similarity can be obtained. The use of generalized equations for fluid properties will be used to reach this goal. In this way it will be shown what the nature of model coolant must be so that the search for such a coolant is narrowed down to a specific range.

Second, those parameters that may be considered unimportant will be omitted and another analysis made. This will lead to a less rigid series of similitude conditions, and thus easier modeling.

II. REVIEW OF THE LITERATURE

The application of dimensional analysis to fluid flow and heat transfer models is discussed by several authors (4, 5, 6, 8, 14, 15, 17, 18, 19, 20, 21, 25, 27, 28, 29). The extent and object of the discussions vary.

Bridgman (4), Klinkenberg and Mooy(17), McAdams (19) and Huntley (14) are interested in modeling as a use for dimensional analysis. Others, such as Murphy (21), Duncan (6), Gukhman (8), Sedov (29), Rayleigh (25) use dimensional analysis as a tool for their discussion of models. Rushton (27), Martin (20) and Johnstone and Thring (15) use dimensional analysis in pilot plant scale-up problems. All of the authors seem to agree that the method of dimensional analysis is a very important and useful tool.

Murphy (21) also goes into the development of prediction equations and the theory of models. Sedov (29) presents many and varied examples of some applications of the modeling technique. Johnstone and Thring (15) place their emphasis on the chemical engineering unit operations. Klinkenberg and Mooy (17) discuss the meanings of the various dimensionless groups in common use, as do several other authors (8, 21, 27, 29).

The Westinghouse Atomic Power Division has used models to study flow patterns and pressure drops in their PWR program (1, 2, 3, 12, 13). The pressures and temperatures of the studies have been about those that are expected in the prototype reactor, and the coolant has been water. The only variable parameters have been the velocity and the pressure drop. The only restriction on the Reynolds number was that the flow be turbulent.

Allis-Chalmers performed some experimental work on a mock-up on a portion of one superheater element for the Pathfinder Reactor (22). The object of the work was to correlate the data for an annular passage containing steam which is being heated from both sides to data for such a passage in which steam is being heated from one side only, and to determine friction factors. This work was mainly an extension of the work done by Heineman (11), who worked with tubular channels heated from one side. The results from the experiments were in agreement with Heineman's work, and were used in the design of the Pathfinder (30). As in the Westinghouse work, the temperature and pressure conditions of the mock-up were the same as for the reactor, and the coolant was steam, as in the reactor. The length scale was unity.

Some numbers are necessary for describing the coolant fluid in both the model and in the prototype. This can be a source of difficulty since there is a wide variance in the published literature for several of the properties of superheated steam. Hawkins (9) reports that there is as much as a 25% difference in the published values for superheated steam. There are two reports that attempt to correlate the data of the several experimentors on the thermal properties of steam at high temperature and pressures (7, 23). The problem is difficult for steam, and worse for any other coolant that might be considered for use in a model. The best that can be suggested for a very general approach to the problem is to use a reference such as Touloukian (31) for specific coolants, or Reid and Sherwood (26) for general approximation relationships.

III. MODEL DEVELOPMENT

A. Dimensional Analysis

The object of dimensional analysis is to reduce the number of parameters to be studied in a given system. The various parameters are grouped into independent dimensionless groups, according to the Buckingham Pi Theorm (14, 15, 21). This method of analysis can give misleading results if every variable that significantly affects the system is not considered. Conversely, the results can be trivial if too many variables that have only a negligible effect are used (14, 15).

The Buckingham Pi Theorm states that if there are "n" parameters, and "m" fundamental dimensions, there are "n-m" dimensionless and independent groupings of the parameters required to express a relationship among the variables. These dimensionless quantities are usually called Pi terms.

One of the benefits of using dimensionless groups is that the results are independent of the measuring system, as long as the system is consistent. This is the principle of similarity. The ratios of the quantities denoting two different values of a variable must remain the same when the absolute magnitude of the unit is changed. Thus, temperatures must be expressed on the absolute scale, rather than on the Centigrade of Fahrenheit scales (15).

One of the points of conflict among several persons in the dimensional analysis field is the question of primary quantities. Are there 3, 4, 5, 6, or more primary dimensions (14, 15, 21)? By the use of dimensional constants, such as $g_c = ML/FT^2$ (dimensionally), the number of variables and the number of fundamental dimensions in the system are both increased by one. Thus, there is no net change in the number of Pi terms. There may be, however, a change in the way that the Pi terms are put together. For example, consider the form of mechanical energy (LF) vs. thermal energy (H). The heat energy dimension is independent of the mechanical energy dimensions only if there is no transition from thermal to mechanical energy (21). This is the case in a heat exchanger, but not in a turbine.

A very interesting problem has been brought up. How does the analyst determine what the variables in the problem are, which are the significant ones, and what are the significant dimensions? The common answer to this question is that the analyst must use experience and judgment. Generally, mechanisms are known, but the analytical expressions are too difficult to formulate or solve. Thus the experimental approach is used to get design data. Only the items in the analytical formulation of the problem need to be considered. Other than this "a priori" knowledge, physical intuition must be used (14).

B. Similitude

Practical use is made of similitude through models. A model is an experimental system so related to the physical system that the observations on the model may be used to describe accurately the performance of the physical system within the similarities between the two. The physical system for which the predictions are formulated from the model is called the prototype (21). The object of the model is to give the desired relationships quickly and cheaply. By making the model much smaller than the prototype, this goal can be reached (13). Also, where the prototype is a multiple structure made up of substantially identical elements, the model may be a model element (2, 15, 22), if the relation between elements is not desired.

With models, the word "shape" has an important meaning. It means not only the geometrical proportions, but also fluid flow patterns, temperature gradiants, and force fields. The configuration of a physical system is determined by the ratios of magnitudes within the system, and does not depend upon the size or nature of the dimensions in which those magnitudes are measured (15). This is the same as has been mentioned as a benefit of dimensionless groups.

The three states of similarity of interest are geometrical, mechanical, and thermal similitude. Very often the various forms of similarities are not compatible, while they seem to be theoretically possible (20). There is only one consistent set of scales that can be determined that will fit all the parameters that could possibly enter into a system. This may be relaxed if some parameters are considered unimportant and an adequate similarity exists.

Geometric similarity means that for every point on the prototype, there exists a corresponding point on the model. Generally this is the easiest condition of similarity to meet because it involves only the length dimension. Mechanical similarity includes static-force similarity, kinematic similarity and dynamic similarity, the last two forms being of interest. Kinematic similitude introduces the time dimension, which in this problem involves a moving fluid system. Thus, geometrically similar systems can be kinematically similar if the flow patterns are similar with respect to geometry and time. Dynamic similitude is concerned with maintaining a similarity between the forces which accelerate or retard moving masses in dynamic systems. The main forces of this type are gravitational, friction, pressure and inertial. Kinematic similarity entails dynamic similarity.

Thermal similarity involves the flow of heat. The four possible mechanisms are radiation, conduction, convection and bulk movement of matter through pressure gradients. Thus, if the system is geometrically similar, thermal similitude requires that corresponding temperature gradients bear a constant ratio to one another, and if the system is moving, kinematic similarity be maintained (15).

C. The Prediction Equation

Since it is desired to relate the experimental data from the model to the prototype, a method of expression is necessary. This method of expression is called the "Prediction Equation". Essentially, every physical equation which is dimensionally homogeneous can be expressed in the form of a relation between dimensionless groups, Pi terms. Every complete physical equation is either dimensionally homogeneous or is capable of being resolved into two or more separate equations that are dimensionally homogeneous.

Consider the following case. For six variables, expressed using three fundamental dimensions, the function can be expressed as

$$\alpha = f(a_1, a_2, a_3, a_4, a_5) \qquad (1)$$

where "α" is the dependent variable, "f" designates "function of", and "a_i" are the independent variables. The variables of the system are assembled into the Pi terms, which are then in turn used to form a functional relationship between the dependent and independent terms. The Buckingham Pi Theorem says that three dimensionless and independent groups can be formed from the above, $n - m = 3$. The function (equation 1) rewritten is

$$\pi_1 = f(\pi_2, \pi_3) \qquad (2)$$

where π_1 contains α and the necessary a_i to make it dimensionless, and π_2 and π_3 are two more dimensionless and independent groups formed by the set of a_i.

D. Modeling

Four basic types of models are possible, and each has its advantages. These four types are true, adequate, distorted and dissimilar models.

A true model has all of the significant characteristics of the prototype reproduced to the required scale. In many cases, a true model is constructed by making the model identical with the prototype in size and structure, but putting much more control and instrumentation into the model than would be permitted in the prototype. If it is only similar, then it must be similar in all respects. It should be possible to predict all or most of the characteristics of the system for which it was developed.

An adequate model is one that is designed to predict only a few selected characteristics of the prototype. It is generally easier to build and operate than a true model, but it does not have the prediction flexibility of a true model. Thus, a model element is not as useful as a complete model, but it can give the data necessary for the design of the element.

The distorted model is one in which some design condition is not satisfied, and the similitude is disrupted so that the prediction equation requires some correction. In view of the fact that thermodynamic and dynamic similarity can not always be met simultaneously, distorted models are common where these are required, especially when the length scale is other than unity.

Some consideration should be given to the distorted model because there is additional analysis involved due to the distortion. The prediction equation (2) can be rewritten as

$$\pi_1 = f(d, \pi_2, \pi_3) \qquad (3)$$

where "d" is the distortion factor, and is dimensionless. To what extent the distorted prediction equation (3) is affected by the distortion is available only by experimental comparison, ideally with the prototype. Since, if the prototype is available, there would be no need for the model, and thus no model, the usual comparison is a second model. The relation between the prototype and one distorted model should be predicted by the relation between two distorted models.

In a model where there is geometric similarity, but either thermodynamic or dynamic similarity must be sacrificed, the use of several fluids will alter the thermodynamic similitude so that several models are available from one set of experimental equipment.

By definition, the prediction factor "δ" is the ratio of the prediction equation of the prototype to that of the model.

$$\delta = \frac{\pi_{1p}}{\pi_{1m}} = \frac{f(\pi_{2p}, \pi_{3p})}{f(\pi_{2m}, \pi_{3m})} \tag{4}$$

The distortion may be in one of the Pi terms for the model, or in several, such that the Pi term is not equal to the corresponding Pi term in the prototype.

Dissimilar models have no apparent relation to the prototype. They usually operate through analogies, such as the common analogy between heat transfer and electrical potentials, or the use of analog computers to express time dependent solutions to a differential equation.

E. Common Dimensionless Groups

In general, hydraulic models are affected by system dimensions, control distance and outline dimensions, fluid properties, density, viscosity and surface tension, and applied forces, acceleration or gravity. One would generally like to predict either the pressure or velocity at all points in the system, since there is a relation between the velocity and pressure. The common dimensionless groups used are the Reynolds, Froude, and the Weber numbers.

The Reynolds number is commonly used whenever the flow regime is considered, such as turbulent or laminar flow. Essentially, the Reynolds number is a quantitative measure of the relative values of the momentum fluxes due to convective and molecular mechanisms. The kinematic viscosity is the physical constant characterizing the molecular transfer. Stated in another way, the Reynolds number characterizes the ratio of the inertia forces acting on an element of fluid to the viscous forces acting on the fluid (8, 21).

If the Reynolds number is high, the effect of the viscous forces is small, relative to inertia, and thus may be insignificant. Where several forces are operative, consideration of the viscous force may be neglected while studying another larger force, and the emphasis shifted. An example is at very high Reynolds numbers, the Euler number is more important (1).

The Froude number is the ratio of the inertia force to the gravitational force. For gases, and flow in horizontal pipes, the gravitational forces are insignificant (6, 14, 21). Exceptions arise where there is a large density change.

The Weber number is important where two phases are in contact. It is the ratio of the inertia force to the surface tension force, and would have a significant effect in a boiling region, or where one liquid phase was surrounded by another.

The Cauchy number is the square of the Mach number and is the ratio of the inertia force to the forces of compression. Since the Mach number is not important until it is close to unity, the compressibility need not be considered until the Mach number is about 0.1 (21). This is such that it can be neglected in reactor work.

In thermal models, the common dimensionless groups are the Prandtl, Nusselt, Peclet, Stanton and Reynolds numbers. These numbers are not all independent. The Peclet number is the Reynolds number times the Prandtl number, and the Nusselt number is the Reynolds number times the Prandtl and Stanton numbers. In each group, one of the numbers has to be omitted for the sake of independence.

The Peclet number is the potentiality for heat transfer (21), while the Prandtl number is the ratio of the heat absorbed by the coolant to the heat transferable through the boundary walls. The Reynolds number indicates how well the fluid is mixed, bringing the cool interior fluid out to the hot walls (17, 27, 28).

The Nusselt group is the ratio of the heat transferred by convection to that transferred by conduction. It is also looked upon as the ratio of the characteristic length to the laminar layer thickness (17). The Stanton number, when multiplied by the temperature difference between the fluid and the wall, is the ratio of the heat actually transferred to that virtually transferable if temperature equalization were permitted.

The best approach seems to be to retain the Nusselt and

Prandtl numbers, in conjunction with the Reynolds number. There will be both conduction, and a slight laminar sublayer. The measure of this sublayer can be important. Also, since the groups are not independent, the ones not known can be determined. The most common expression on convective heat transfer correlations are usually given in terms of the Nusselt, Prandtl, and Reynolds numbers, and this convention allows for easier correlation.

IV. THE PATHFINDER REACTOR

The Pathfinder Boiling Water Integral Nuclear Superheating Reactor is presently being completed by the Atomic Energy Division of the Allis-Chalmers Manufacturing Company, Milwaukee, Wisconsin, for the Northern States Power Company, Minneapolis, Minnesota. The plant is located near Sioux Falls, South Dakota.

The reactor is a 600 psi, thermal, light water moderated unit. Forced circulation is used in the UO_2 fueled vertical, annular boiler core. The steam is separated, dried and passed down through the internal superheating core, exiting at from 725 to 675 ^{o}F. Diagrams of the reactor and of the core configuration are shown in figures 1 and 2 (30).

The active boiler flow is 57,400 GPM, producing 616,125 lb/hr of steam at 489 ^{o}F and 600 psig. The void fraction is 45.5% at the top of the boiler. Each boiler element is a boxed array of 81 rods set of a 0.535 inch square pitch. For more details, see tables 1 and 2. There is no flow redistribution among elements once the coolant has entered an element. There is flow redistribution within an element. The effect of boiling on flow resistance needs to be determined for burn out studies.

An increased boiling causes increased flow resistance, causing less flow at a given pressure, and thus resulting in

Table 1. Boiler core and fuel element details

Fuel element assemblies:		
Bundle size @ room temperature	4.735 sq. in.	
Number of fuel rods per bundle	81	
Bundle geometry	9 x 9 array	
	0.535 in. square pitch	
Rod section per 6 ft. rod	4	
Box size	5 in. square x 99 in. long	
Fuel material	UO_2 @ 10.41 gm/cc.	
Clad and box material	Zircaloy-2	
	Lower Section	Upper Section
Pellet diameter	0.348 in.	0.310 in.
Clad thickness	0.028 in.	0.026 in.
Rod diameter @ room temperature	0.408 in.	0.367 in.
Core:		
Mean outside diameter	69.0 in.	
Mean inside diameter	31.3 in.	
Active length	72 in.	
Volume	123 $ft.^3$	
Number of fuel elements	96	
Number of cruciform control rods	16	

Table 2. Boiler core operating parameters

Full power	157.4 Mwt.
Full flow (active)	57,400 GPM
Inlet subcooling	3.8 Btu/lb.
Steam production	616,125 lb/hr.
Exit voids	45.5%
Pressure drop	13.6 psi.
Inlet coolant velocity	13.6 ft/sec.
Maximum surface clad temperature	514 °F.
Maximum fuel temperature	4215 °F.
Average power density	1280 Kw/ft^3.
Average specific power	24.0 Kw/Kg of U
Average heat flux	122,000 $Btu/hr/ft^2$.
Maximum heat flux	447,000 $Btu/hr/ft^2$.

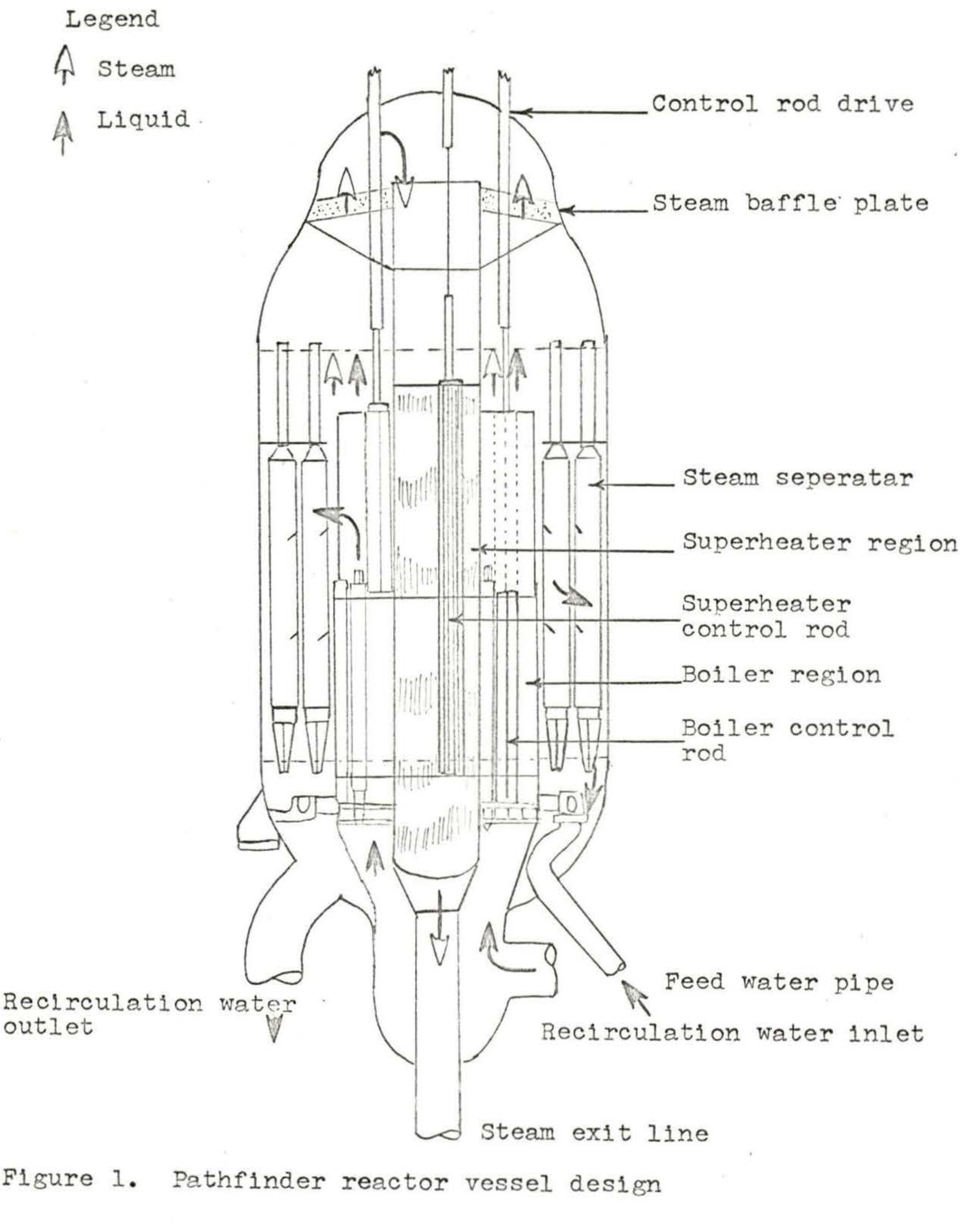


Figure 1. Pathfinder reactor vessel design

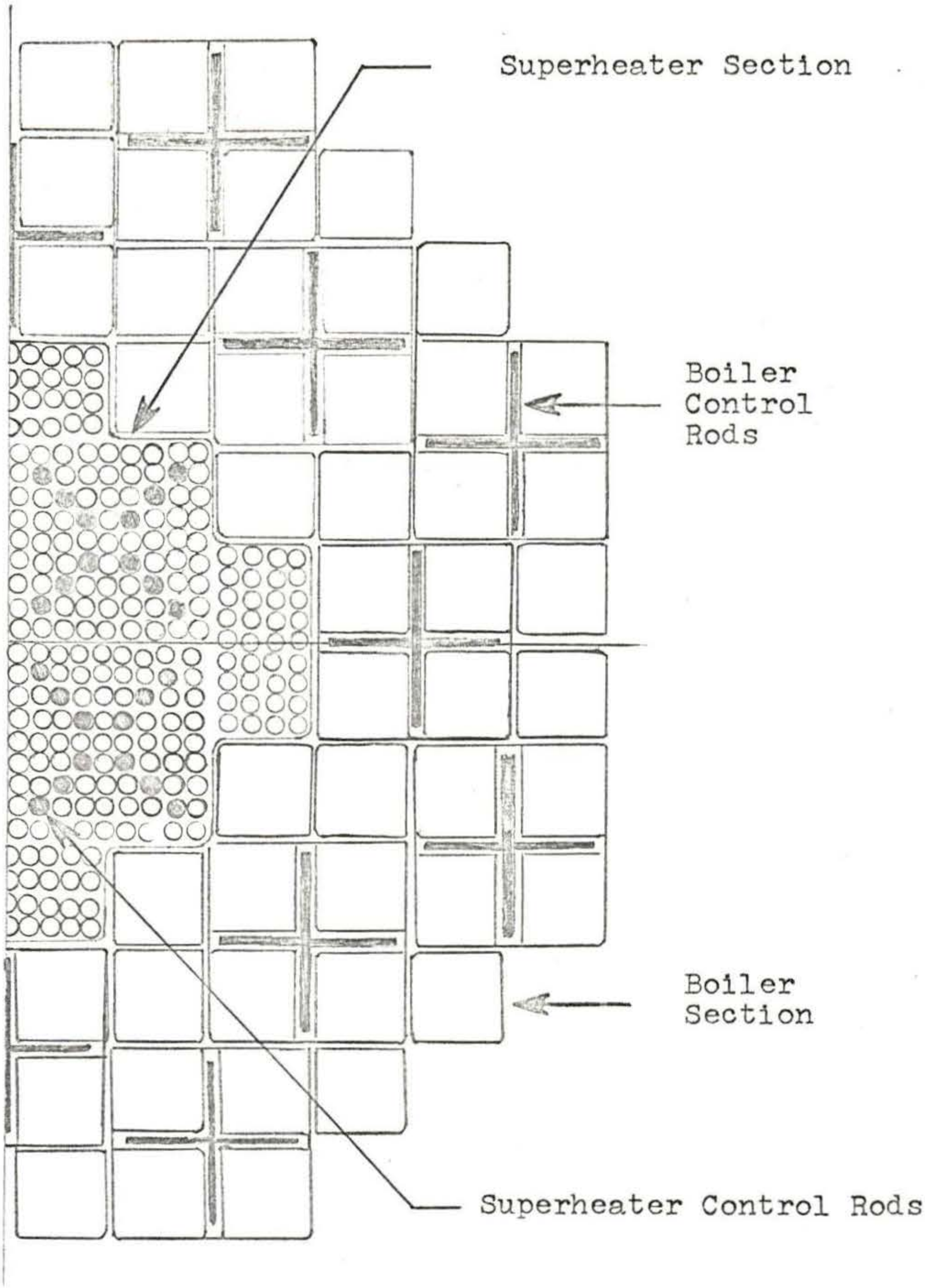


Figure 2. Cross section of Pathfinder core

more boiling. The inherent feed back in a light water moderated system makes this a case for transient analysis. To this end, one of the interesting items studied for the design of the boiler was the dependence of pressure drops on core operating parameters, primarily the effect of boiling.

Some of the variables which affect the amount of boiling are core power level, core inlet subcooling, core pressure, and axial power distribution. Many of these variables are interrelated. It is the outlet pressure that controls the boiling temperature, but it is the inlet pressure that is controlled by the pumps. The pressure drop is the flow driving force, and the power level controls the amount of boiling, for a given flow rate. The axial power distribution controls where the boiling will take place, relative to the different regions in the core. Since there is no redistribution within the core, the major problem is the flow resistance within an element, which leads to the distribution among the elements.

The analysis of the boiler problems was made using established computer codes, which have been compared to many earlier experimental tests by those who developed the codes. There was no direct tests for the Pathfinder boiler core. However, for the superheater, an experimental program was carried on by Allis-Chalmers.

The superheater design details are given in tables 3

Table 3. Superheater core and fuel element dimensions

Fuel element assemblies:		
Outside diameter of element @ room temperature		1.076 in.
Fuel tubes per bundle		2
Total tubes per element		4
Tube geometry		concentric
Poison rod diameter		0.467 in.
	Outer Diameter	Inner Diameter
Outer insulating tube	1.076 in.	1.024 in.
Inner insulating tube	0.967 in.	0.937 in.
Outer fuel tube	0.839 in.	0.769 in.
Inner fuel tube	0.630 in.	0.560 in.
Core:		
Mean diameter		30 in.
Active length		71.5 in.
Volume		32.5 ft^3.
Number of fuel elements		412
Number of cylindrical control rods per cluster		12
Number of control rod clusters		4
Flow area		0.993 ft^2.
Heat transfer area		1817 ft^2.

Table 4. Superheater operating parameters

Thermal power generated	31.5 Mwt.
Active flow	608,935 lb/hr.
Average inlet velocity	128 ft/sec.
Average outlet velocity	212 ft/sec.
Core pressure drop	61 psi.
Maximum fuel cladding temperature	1270 ^{o}F.
Average power density	970 Kw/ft^3.
Average heat flux	59,100 Btu/hr/ft^2.
Maximum heat flux	219,000 Btu/hr/ft^2.
Inlet steam temperature	489 ^{o}F.
Average outlet steam temperature	725 ^{o}F.

and 4, and figure 3a. The saturated steam from the boiler passes down through several annular passages. The pressure drop in the superheater is about 4.5 times the pressure drop in the boiler, while the thermal power is about one fifth of the boiler thermal power.

Some of the annular passages heated the steam from both sides of the annulus. The literature covered internal heating of annular passages and there was some doubt that this information could be directly used for the design of the Pathfinder. To verify that the literature information was useful, or that it did not apply, a model of the center coolant passage of a superheater element was made and tested by Allis-Chalmers (22). The length scale was unity, and the reactor conditions were used. Thus there was no problem with thermodynamic or dynamic distortions. The only significant difference was the power distribution in the test element. It was flat as compared with the Pathfinder reactor, which was peaked.

Some of the facilities of the experimental system used by Neusen et al. (22), for Allis-Chalmers, were extensive. The d-c power available to simulate nuclear heating was 1.5 megawatts. The system was rated at 1500 psi and 1200 ^{o}F. Saturated steam could be produced at up to 900 lb/hr, at 600 psig.

The small gap (0.060 in.) and the large diameter (0.878 in. ID) of the annulus tends to make the annulus

Table 5. Comparison of superheated steam experiments used by Allis-Chalmers (30)

Experimenter	McAdams	Heineman
Test assembly:		
Length (in.)	12.0	40 to 80
Annulus D_i (in.)	0.250	0.000
D_o (in.)	0.382	0.666
D_{eq} (in.)	0.131	0.666
Maximum L/D obtained	57	6 to 121
Conditions:		
Pressure (psi.)	115 to 3500	300 to 1500
Temperature ($^{o}F.$)	430 to 1000	550 to 900
Reynolds number ($x\ 10^{-3}$)	7 to 40	60 to 370
Heat flux (Btu/hr/ft.^{2} x 10^{-4})	5.6 to 7.2	5.00 to 28.7

Collier and Lacy			Neusen et al.
A	B	C	
29.6	29.6	29.6	72
0.375	0.375	0.375	0.875
0.553	0.553	0.886	0.995
0.178	0.178	0.241	0.105
151	151	112	685
155 to 1075	20	20	600
220 (superheat)	300 (superheat)	300 (superheat)	489 to 1200
130 to 320	34 to 64	26 to 44	20 to 140
5.10 to 12.8	0.62 to 1.0	0.5 to 1	2.0 to 14

Table 6. Results of various superheated steam experiments

McAdams:

$$(Nu)_f = 0.0214(Re)_f^{0.8}(Pr)_f^{1/3}(1 + 2.3(L/D_{eq}))$$

Comments: Low L/D, low Re, and a sudden turn in the flow stream, just upstream from the test section was a possible complicating factor.

Collier and Lacy:

(a) all test combined,

$$(Nu)_f = 0.0058(Re)_f^{.945}(Pr)_f^{1/3}(L/D_{eq})^{-0.1}$$

$$(Nu)_f = 0.0035(Re)_f^{.945}(Pr)_f^{1/3}(1 + 4.56D_{eq}/L)$$

Comments: Low L/D_{eq}, weighted by low pressure results, effect of annular geometry not isolated.

(b) high pressure results,

$$(Nu)_f = 0.0357(Re)_f^{0.8}(Pr)_f^{1/3}(L/D_{eq})^{-0.1}$$

$$(Nu)_f = 0.0208(Re)_f^{0.8}(Pr)_f^{1/3}(1 + 8.97D_{eq}/(L + 12D_{eq}))$$

Comments: Low L/Deq, effect of test section geometry not isolated.

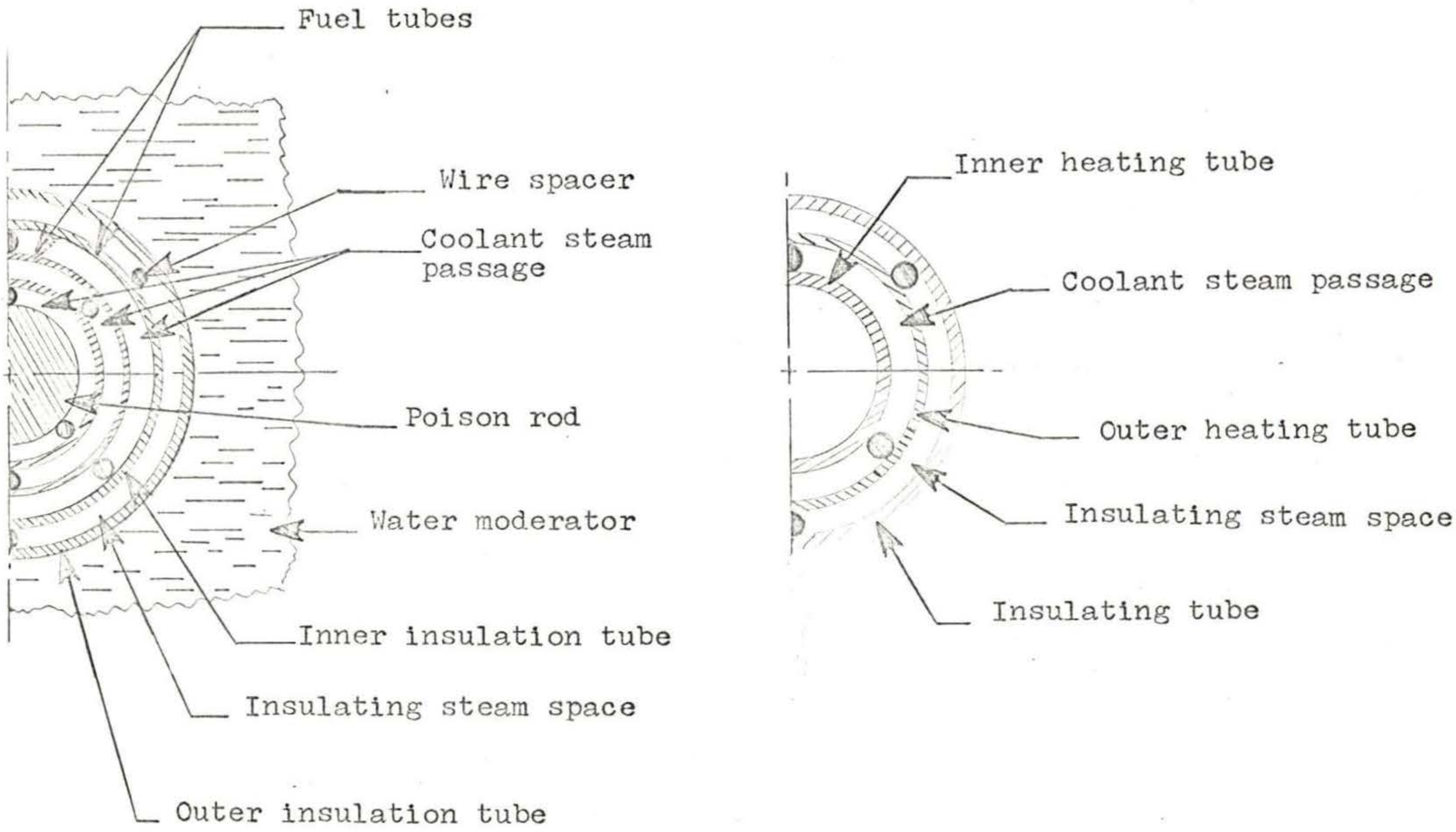
Heineman:

$$(Nu)_f = 0.0157(Re)_f^{.84}(Pr)_f^{1/3}(L/D_{eq})^{-0.04} \qquad 6<L/D_{eq}<60$$

$$(Nu)_f = 0.0133(Re)_f^{.84}(Pr)_f^{1/3} \qquad L/D_{eq}>60$$

Neusen et al.:

Heineman's correlation of Nusselt numbers for L/D_{eq} appears appropriate in a thin annulus with both walls heated. The measured film coefficients are about the same on both the inner and outer walls. The L/D_{eq} effect is negligible for the large L/D_{eq} investigated. Annular geometry with longitudinal wire spaces is amenable to treatment with the equivalent diameter concept.



(a) (b)

Figure 3. (a) Reactor superheater element and (b) Allis-Chalmers test element

approach the case of parallel plates. The wire spacers separated the flow into three channels, but the standard equivalent diameter rationalized the data very well. As expected from such a case, where the spacer wire served to keep an equal potential in both tubes, the convective heat transfer coefficient was almost equal from both walls due to the approximately equal heat generation in each tube. The heat generated in the wires was less than 2% of that generated in the tubes.

Of particular interest was the applicability of the classical Nusselt number for a thin annular passage spaced with longitudinal spacers. The comparisons made with other experimental work is shown in tables 5 and 6. Note the L/D, Reynolds number, and operating conditions. These are the reasons for doubting the applicability of the literature work to the Pathfinder. Figures 3a and 3b show the relation between the reactor and the model elements.

The results of the work did show that the work of Heineman, while neglecting geometric similarity, was adequate for predicting the Nusselt number for the Pathfinder superheater. The standard equivalent diameter rationalized the data well. This standard diameter is four times the flow area (annular area less the area of the three spacer wires) divided by the total perimeter (annular walls plus the three spacer wires). However, could this work have been performed at lower pressures and temperatures?

Can another fluid other than water vapor be used, and give reasonable results? What are the requirements of a true model, and if a true model is impractical, can an adequate small model be designed?

V. THE MODEL ANALYSIS

A. Parameters and Dimensionless Groups

The model study was undertaken to develop a method of predicting many of the thermal characteristics of the prototype element used by Allis-Chalmers to obtain their design data for the Pathfinder. First, the conditions for a true model will be determined, taking into account all variables. Second, those variables not significant for an adequte thermal model will be neglected and a series of new design conditions determined, consistent for the variables selected.

Many parameters enter into the Pathfinder reactor system. A list of these variables is given in table 7. Six dimensions are considered; force, thermal energy, mass, length, time and temperature. These six were reduced to four; mass, length, time and temperature, through the use of the following dimensional formulas,

$$F = MLT^{-2} \tag{5}$$

$$H = FL = ML^{2}T^{-2} \tag{6}$$

and are included in table 7.

The variables are grouped into the necessary dimensionless groups, Pi terms, and are presented in table 8. The Pi terms were determined based on the M, L, T, θ system, and using the algebraic method for the determination of the exponent for each parameter in the group. The independence of each group is assured by permitting only a selected few

Table 7. Parameters to be studied

Parameter	Symbol	Dimensions H,F,M,T,L θ system	Dimensions M,L,T, θ system
1. Tube length	L	L	L
2. Equivalent diameter	D_{eq}	L	L
3. Annular spacing	ΔD	L	L
4. Effective roughness	ϵ	L	L
5. Fluid bulk velocity	u_b	LT^{-1}	L^{-1}
6. Gravity	g	LT^{-2}	LT^{-2}
7. Fluid inlet usaturation	S	HM^{-1}	L^2T^{-2}
8. Fluid density	ρ	ML^{-3}	ML^{-3}
9. Fluid viscosity	μ	$FL^{-2}T$	$ML^{-1}T^{-1}$
10. Fluid thermal conductivity	k	$HL^{-1}T^{-1}\theta^{-1}$	$MLT^{-3}\theta^{-1}$
11. Fluid specific heat	C	$HM^{-1}\theta^{-1}$	$L^2T^2\theta^{-1}$
12. Fluid inlet bulk temperature	t_i	θ	θ
13. Change in fluid bulk temperature	Δt_L	θ	θ
14. Difference between bulk and wall temperature	Δt_d	θ	θ
15. Fluid critical temperature	t_c	θ	θ
16. Fluid inlet pressure	P_i	FL^{-2}	$ML^{-1}T^{-2}$
17. Fluid pressure drop	ΔP	FL^{-2}	$ML^{-1}T^{-2}$
18. Fluid critical pressure	P_c	FL^{-2}	$ML^{-1}T^{-2}$
19. Convective heat transfer coefficient	h	$HT^{-1}L^{-2}\theta^{-1}$	$M\ T^{-3}\theta^{-1}$
20. Heat flux	Q	$HT^{-1}L^{-2}$	MT^{-3}

Table 8. Pi terms

Pi Terms	Symbolic Form	Common Name (if any)
π_1	hD_{eq}/K	Nusselt number
π_2	$D_{eq}\rho u_b/\mu$	Reynolds number
π_3	$C\mu/K$	Prandtl number
π_4	$\Delta P/\rho u_b^2$	Euler number
π_5	u_b^2/gL	Froude number
π_6	$\Delta t_i/t_c$	Reduced temperature
π_7	$\Delta P_i/P_c$	Reduced pressure
π_8	u_b^2/CDt_L	Eckert number
π_9	u_b^2/S	
π_{10}	L/D_{eq}	
π_{11}	$\Delta D/D_{eq}$	
π_{12}	$Q/D_{eq}K\Delta t_d$	
π_{13}	$\Delta P/P_i$	
π_{14}	$\Delta t_L/t_i$	
π_{15}	$\Delta t_d/t_i$	
π_{16}	ϵ/D_{eq}	

parameters to appear in more than one group.

B. Development of a True Model

The geometric parameters to be scaled are length, equivalent diameter, and annular spacing. These must be scaled proportionally to preserve similarity. If the length scale is taken as

$$L_p/L_m = n \tag{7}$$

where the subscripts "p" and "m" indicate the prototype value and the model value respectively, the scales for the equivalent diameter and the annular spacing are respectively

$$(D_{eq})_p/(D_{eq})_m = n \tag{8}$$

and

$$(\Delta D)_p/(\Delta D)_m = n. \tag{9}$$

The equivalent diameter is calculated subtracting the area of the wire spacer from the annular area, and dividing the difference by the sum of the surface areas of the annulus and the wire spacers, as considered by Allis-Chalmers.

$$D_{eq} = \frac{(D_o^2 - D_i^2) - 3/4\ (D_o - D_i)^2}{D_o + D_i + 1\tfrac{1}{2}(D_o - D_i)} \tag{10}$$

The model will be in the same gravitational environment as the prototype, neglecting any small difference due to different locations on the earth. Thus, the gravitational scale must be unity,

$$g_p = g_m. \tag{11}$$

Now that both the length and gravitational scales have been selected, the time scale has also been fixed,

$$T_p/T_m = n^{\frac{1}{2}}. \tag{12}$$

Using the time and length scale, the velocity scale is

$$(u_b)_p/(u_b)_m = n^{\frac{1}{2}}. \tag{13}$$

Two characteristics in the M. L, T, θ sequence remain to be scaled. Since the critical pressure of water is 218 atm., and that of several possible model fluids is in the range from 40 to 110 atm., the pressure scale can be expressed as

$$(P_c)_p/(P_c)_m = a \tag{14}$$

where "a" varies between 1.8 and 5.1. This now defines a mass scale as

$$M_p/M_m = an^2. \tag{15}$$

The density and viscosity scales are respectively

$$\rho_p/\rho_m = a/n \tag{16}$$

and

$$\mu_p/\mu_m = an^{\frac{1}{2}}. \tag{17}$$

The temperature scale is defined as

$$(t_c)_p/(t_c)_m = b. \tag{18}$$

The Allis-Chalmers system operated with a reduced pressure of about 0.2 and a reduced temperature of about 0.95. This can be considered the ideal gas region, within reasonable error. The compressibility factor for most gases is between 0.9 and 1.0 for these reduced conditions. Therefore, the temperature, pressure and density can be related through the ideal gas law,

$$PA = \rho Rt \tag{19}$$

where "A" is the molecular weight of the gas. When the ratio of the ideal gas law for the prototype to the ideal gas law for the model is taken, and equations 14, 16, and 18 are applied, the ratio reduces to

$$A_m = A_p n/b \tag{20}$$

It may be noted that the pressure scale has no net effect on the molecular weight scale since the pressure scale is canceled by the density scale in the determination of equation 20. The dependence of the length scale on the molecular weight of the fluid chosen and the temperature scale defined by this fluid, through equation 18, can be seen by rearranging equation 20,

$$n = A_m b/A_p. \tag{20a}$$

It may be noted that "b" will lie between 1.0 and 3.0 when water is the prototype fluid. Therefore,

$$A_m/18 < n < A_m/6. \tag{20b}$$

The relationship between "A_m" and "n" is shown in figure 4. The area between the two straight lines is the area of interest defined by equation 20b and the limits on b.

Further, the value of a can be related to A_m through the Licht and Stechert method for the determination of the viscosity of an ideal gas (26). This method relates the viscosity to the temperature, pressure, molecular weight and reduced temperature of the gas. These parameters have all been scaled, and a ratio of the prototype characteristics to the model characteristics can be determined as with the ideal gas law,

$$\mu = 0.000630 \frac{A^{1/2} P_c^{2/3}}{t_c^{1/6}} \frac{t_r^{3/2}}{t_r + 0.8} \tag{21}$$

$$\mu_p/\mu_m = \frac{(b/n)^{1/2} a^{2/3}}{b^{1/6}} \, . \tag{22}$$

However, the left hand side of equation 22 has previously been defined with equation 17. The result of equating equations 17 and 22 is

$$n = (A_p/A_m)\,(ab^{\frac{1}{2}})^{-2/3} \tag{23}$$

The limits on a and b are substituted into equation 23 and the result is

$$12.2\,/A_m \; > n > \; 4.21/A_m \quad . \tag{23a}$$

This result is the pair of curved lines plotted in figure 4. The area within these lines is the area defined by equation 23a.

The intersection of the areas defined by equations 20b and 22b, is the region that satisfies both equations. The values of a and b to satisfy the selected set of A_m and n can be calculated through equations 20 and 22a. It is clear from figure 4 that the limits of a and b place strict limits on the length scale, permitting a maximum value of 1.5. The molecular weight permitted places a severe limit on the choice of fluids acceptable for the model with A_m less than 15.0.

Since the molecular weight must lie between 5.0 and 15.0, the number of fluids that can be considered for use in a true model is limited. Since the Licht-Stechert viscosity determination method is invalid for hydrogen and helium, the logical choice of model fluid is among methane, ammonia, and hydrogen flouride, based on figure 4. Ammonia is a reasonable fit with $b = 1.95$, $a = 1.60$ and $A_m = 17$. Methane has an $A_m = 16$ and the scales a and b equal to 4.70 and 2.95 respectively. While both fluids have an A_m at about the upper limit determined for a true model, the value of n in the region of figure 4 is less than 1.5. Thus, the physical size of the model will be about the same size as the prototype, or much larger.

C. Considerations for an Adequate Model

Upon studying the preceeding information, it is

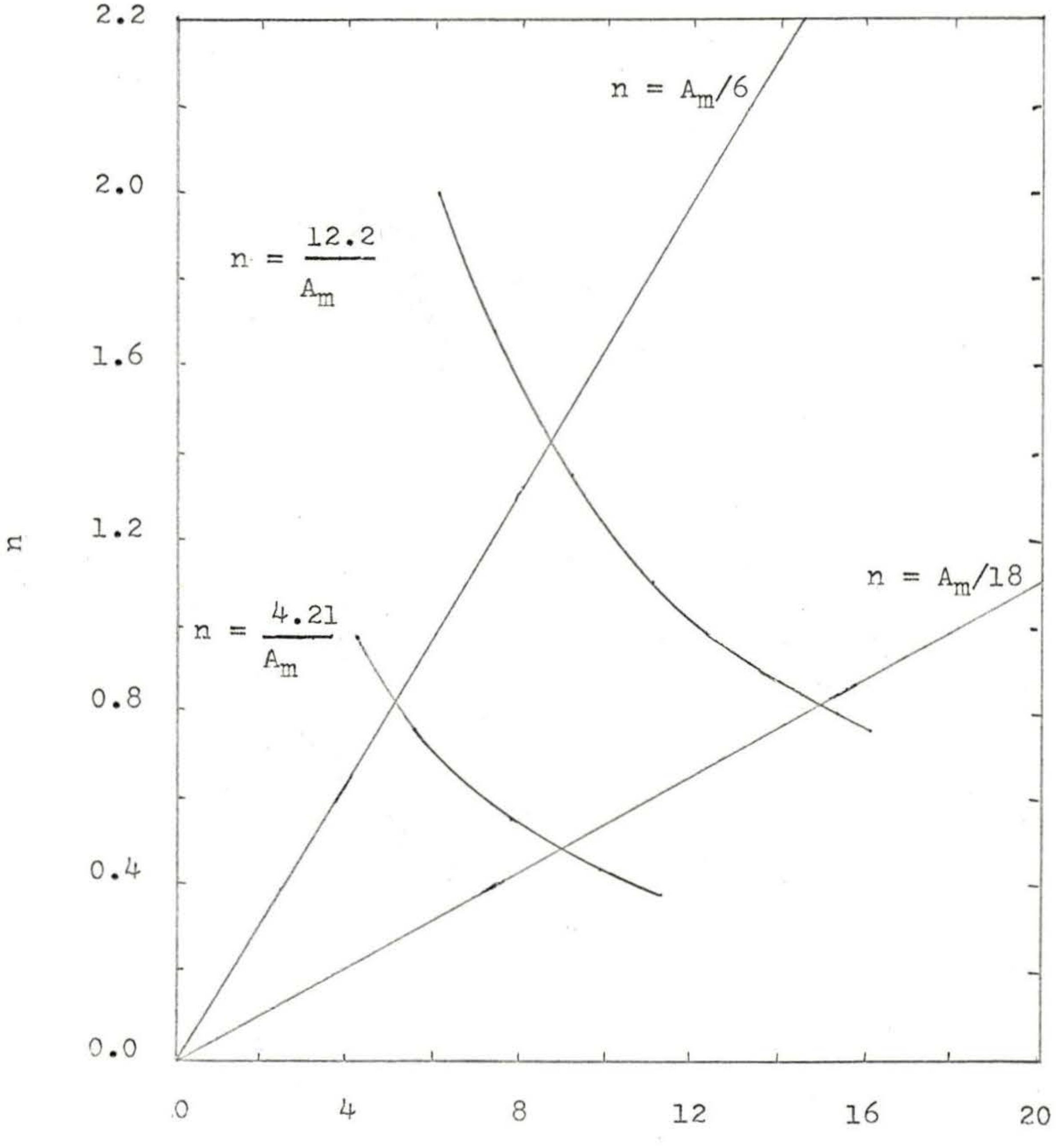


Figure 4. Plot of equations 20b and 23a

desirable to use an adequate model, and study only some selected characteristics. The procedure is to neglect some variables in order to determine a less rigid scale system. The Froude number can be neglected; thus eliminating the time scale determined earlier. This is stating that gravity is not an important factor in a thermal model.

In making the change, it is desirable to place limits upon the model design. The model will be sized so that it is convenient for a small laboratory. The length scale should be about 3. Second, the need for high pressurization will be unnecessary. This requires a pressure less than 10 atm., and close to 1 atm. The system will be required to remain in the ideal gas region of temperature and pressure conditions, and the molecule should have sufficiently simple form.

The size of the annulus will present mechanical problems if the length scale is too large. Also, due to the concern of Allis-Chalmers about the effect of a thin annulus, the ratio of the annular gap to the hydraulic diameter should be kept similar. Among the problem that can exist if the $\Delta D/D_{eq}$ ratio is not maintained is the choice of a wide annulus with a small flow area. An additional problem is the heat generation within large wires.

So that flexibility of the geometric characteristics is available, standard condenser tubing was used to determine suitable pairs of tubes for the model. Reasonable combinations are given in table 9. The range of length scale covered is from n = 2.0 to n = 4.3. An interesting note is that there are only three distinct annular spacings for the several fluids listed, or three distinct annular space scales. The annular space scale should be numerically equal to the length scale, which further limits the selection based on geometric similarity. The annular space scale for the three values of the annular space in table 9 are 2.19, 3.75 and 4.45.

Another consideration is the cross sectional metal area of the tubes. It is desirable to have equal heat fluxes from each side of the annulus. This requires that the metal area of the tube be weighted by the heat transfer area per unit length. Since there is a very small difference between the heat transfer area for the inside and outside tube, the metal area of the outer tube will be slightly less than that for the inner tube. If the outer tube has a larger area, the tube can be ground to a suitable size.

The actual temperature and pressures of the system are not as important as the changes in these parameters. It is sufficient to state that the system is in a temperature and pressure region, such as the ideal gas region for the gas considered. The object of this thermal model is to determine the heat transfer characteristics of the Pathfinder system. Since this is the case, a definition of the temperature and

Table 9. Possible tube combinations for the Pathfinder model

Length Scale	Inner Tube OD in.	Inner Tube BWG	Metal Area in.^{2}	Outer Tube ID in.	Outer Tube BWG	Nominal Size in.	Metal Area in.^{2}	Annular Space in.
1.95	5/8	19	0.077	0.680	20	3/4	0.079	0.0275
2.13	1/2	18	0.0694	0.555	20	5/8	0.065	0.0275
2.20	3/8	18	0.0502	0.430	20	1/2	0.051	0.0275
2.32	1/4	22	0.0195	0.305	20	3/8	0.374	0.0275
2.49	7/8	10	0.312	0.920	8	1 1/4	0.565	0.0275
3.40	1	8	0.364	1.032	12	1 1/4	0.391	0.0160
3.43	3/4	10	0.260	0.782	12	1	0.305	0.0160
3.50	1/2	16	0.0888	0.532	12	3/4	0.220	0.0160
3.60	3/8	18	0.0502	0.407	12	5/8	0.177	0.0160
3.72	5/8	12	0.177	0.657	12	7/8	0.262	0.0160
3.96	7/8	18	0.127	0.902	18	1	0.146	0.0135
4.00	1/2	16	0.888	0.527	18	5/8	0.0888	0.0135
4.22	3/8	18	0.0502	0.402	18	1/2	0.0694	0.0135
4.33	1/4	22	0.0195	0.277	18	3/8	0.0502	0.0135

pressure scales are satisfactory, even if they do not agree with the scales determined for the true model. The changes of these parameters in the model will be directly related to the similar changes in the prototype.

It can be shown that the temperature and pressure dependent properties of fluids that may be considered for the model are also dependent on other items, such as molecular structure. Therefore, no two fluids can be expected to have all of their thermophysical properties follow a rigid scaling system and be similar through several temperature and pressure ranges. Using one geometric model, several fluids permit many systems to be studied by varing more than just the mass and energy transport parameters of velocity and heat flux.

By keeping the same geometric system, and not considering the actual system operating conditions, eleven Pi terms are eliminated. These terms are either constant, such as L/D_{eq}, $\Delta D/D_{eq}$ and ϵ/D_{eq}, or are considered unimportant, such as the reduced temperature pressure, $\Delta P/P_i$, $\Delta t_i/t_i$, and $\Delta t_d/t_i$. The second group depends on the state pressure and temperature. The Froude number was eliminated because there is the gravitation effect on the thermal system can not be studied conveniently. The Eckert number and the ratio u_b^2/S are also omitted since there is no change of thermal to kinetic energy in a simple heat transfer system. The system is now reduced to four thermal Pi terms, including the Reynolds number, for which eight parameters must be scaled.

D. The Adequate Model

To propose an adequate model, one can select definite temperatures and pressures. For the sake of making these conditions convenient to meet, a pressure of 5 atm. and a boiling temperature of about 350 - 400 °K was selected. This puts several common organic fluids within reach. The major difference between this model and the true model is the pressure of the system.

The ideal gas law, its relation to the density scale, and the scale specified by the temperature and pressure relationship above, yields

$$1.8 < t_p/t_m < 2.0 \qquad (24)$$

$$\frac{P_p}{P_m} = 8 \qquad (25)$$

$$\frac{\rho_p}{\rho_m} = \frac{A_p T_p P_m}{A_m T_m P_p} \qquad (26)$$

$$\frac{\rho_p}{\rho_m} = \frac{A_p}{A_m} \quad \frac{1.9}{8} = \frac{1}{4.2} \frac{A_p}{A_m} . \qquad (26a)$$

To determine a usable value of ρ_p/ρ_m and thus what A_p/A_m should be, the Reynolds number may be considered. If the length is scaled by n, then the product $u_b\rho/\mu$ must be scaled by 1/n. By inspection of several possible organic fluids, such as ethyl alcohol, acetone, or freon, the viscosity scale is about 2,

$$\frac{\mu_p}{\mu_m} \approx 2. \qquad (27)$$

Thus, the value of $(u_b)_p/(u_b)_m$ is expressed as

$$(u_b\rho)_p/(u_b\rho)_m \approx \frac{2}{n} . \qquad (28)$$

The value of $(u_b)_p/(u_b)_m$ is

$$\frac{(u_b)_p}{(u_b)_m} = \left(4.2\ \frac{A_m}{A_p}\right)\left(\frac{2}{n}\right). \qquad (29)$$

For water as the prototype fluid,

$$\frac{\rho_p}{\rho_m} = \frac{4.3}{A_m} \qquad (26b)$$

$$\frac{(u_b)_p}{(u_b)_m} = \frac{A_m}{2.15n} \qquad (29a)$$

which are shown in figure 5.

The plot in figure 5 shows the above two equations for various values of A_m and four values of n. The velocity ratio increases with increasing A_m and decreasing n. Thus, if an organic gas is used, the velocity scale appears to be very attractive. The density scale is not a function of the length scale and at high A_m, it is fairly insensitive to A_m.

For a specific coolant, the thermophysical properties are scaled by the selection of a model fluid. Perry (24) gives some general rules for estimating the value of the

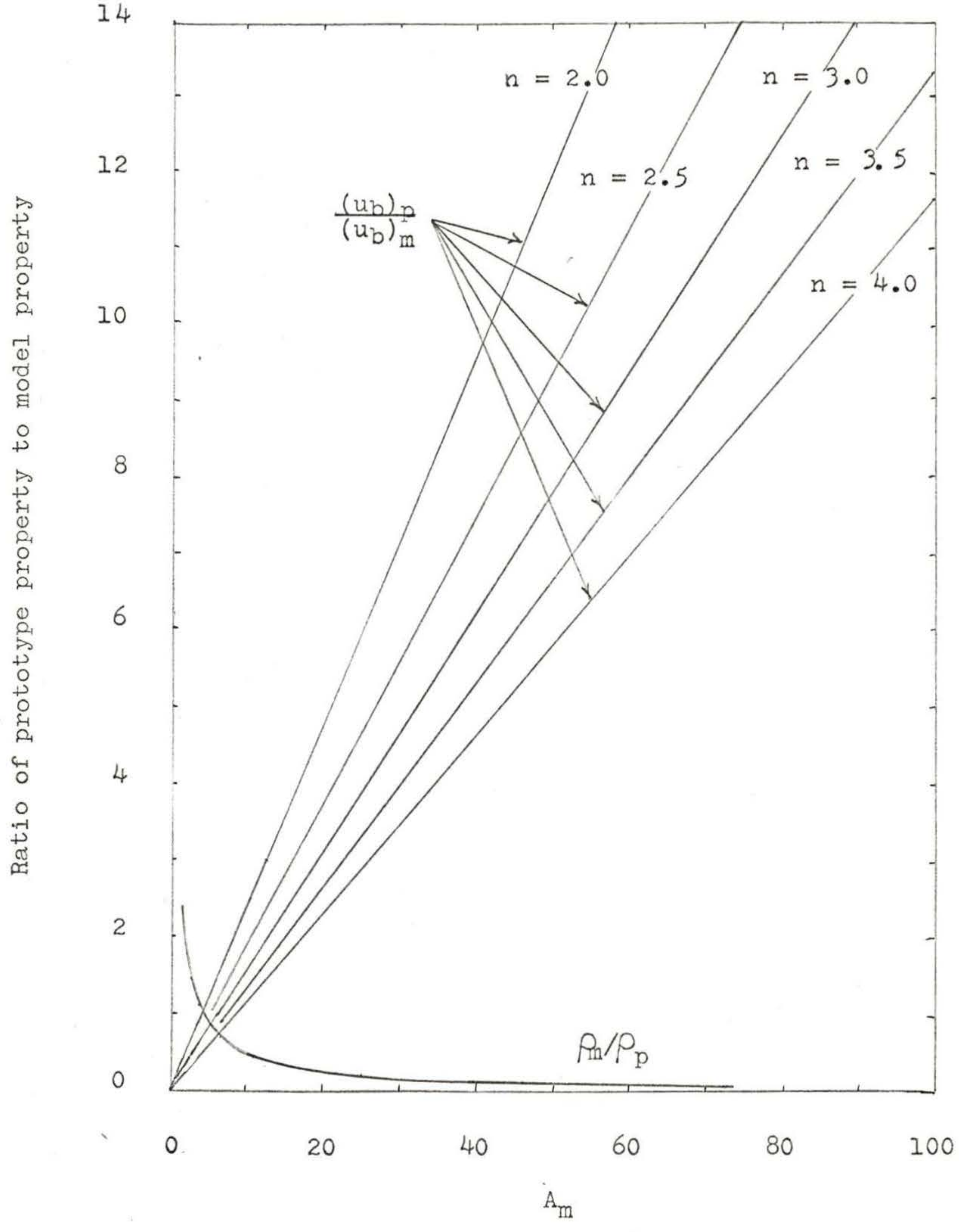


Figure 5. Plot of equations 26b and 29a

Prandlt number for different types of molecules. By limiting the selection of model fluids to those which have the same Prandlt number as water, this group will maintain similitude. However, the equality of the Prandlt number does not mean that the individual parameters vary according to the proper scale. An inspection of tabulated Prandlt numbers in Reid and Sherwood (26) indicates that the previously mentioned gases are satisfactory.

If the viscosity determination method of Licht and Stecher (equation 21) is applied, using the actual model condition, the relation will give a value for A_p/A_m. In this particular situation ethanol has a good fit for the model conditions of temperature and pressure.

VI. CONCLUSIONS AND RECOMMENDATIONS

The design conditions for models of a heat transfer system such as that associated with the Pathfinder reactor have been developed and their practicability studied. It was determined that a true model is not practical and an adequate model has been proposed, using some medium weight organic fluids. For a length scale of about 3.5, the velocity scale will be about 6.7 if the molecular weight of the gas is 50, and 13.5 if the molecular weight is 100. The range of fluids from ethol alcohol to the light freons appear very promising. The two items to be stressed are the ideal gas conditions and the fact that the model can not be formed into a true model.

It was shown that a true model was practical only for a very low molecular weight gas and for a length scale less than unity. Thus, the size of the model would actually be greater than the prototype, which is contrary to the reason for this model study. The goal was to reduce size and power requirements.

It is proposed that an adequate model be built and experiments performed. This data should then be correlated to the previous steam experiments. Possibly, several experimental assemblies could be used to gather data for several different length scales. The effect of the annular space to equivalent diameter ratio should be very interesting as it was a serious question to Allis-Chalmers and was not totally resolved since the length scale of unity in all respects permitted a direct application of their data to the Pathfinder design.

In addition, the possibility of using additives or mixtures to vary the physical properties of the fluid should not be overlooked. This will be necessary if an attempt is made to include the boiler portion of the reactor in a model with the superheater.

VII. LIST OF SYMBOLS USED

A	Molecular weight
C	Specific heat
D_{eq}	Equivalent diameter
D_i	Inside diameter
D_o	Outside diameter
F	Force dimension
H	Heat dimension
L	Length dimension
M	Mass dimension
Nu	Nusselt number
P	Pressure
P_c	Critical pressure
P	Pressure drop
Pr	Prandtl number
Q	Heat flux
Re	Reynolds number
S	Gas unsaturation
T	Time dimension
a	Temperature scale
b	Pressure scale
d	Distortion factor
f	Designates function
g	Gravity

h	Convective heat transfer coefficient
k	Thermal conductivity
n	Length scale
t_c	Critical temperature
Δt_d	Temperature difference between wall and gas
Δt_l	Temperature increase of gas
Δt_i	Inlet gas temperature
u_b	Gas bulk velocity
δ	Prediction factor
ϵ	Tube roughness
θ	Temperature dimension
μ	Gas viscosity
π	Designates Pi term
ρ	Gas density

Subscripts

f	Properties evaluated at film temperature
m	Model value of parameter
p	Prototype value of parameter

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